

LTU UNIVERSITY – SOFIA
FACULTY OF FOREST INDUSTRY
DEPARTMENT OF MECHANICAL ENGINEERING AND AUTOMATION
OF PRODUCTION

STEFAN HRISTOV STEFANOV

Associate professor, PhD,
mechanical engineer, engineer-mathematician

**INTEGRATION OF DAMAGE DIFFERENTIALS (IDD)
FOR FATIGUE LIFE ASSESSMENT UNDER ANY LOADING**

AUTHOR'S SUMMARY

**OF
THESIS**

presented for conferring DSc degree on the author

Sofia, 2011

FOREWORD

The phenomenon of **fatigue of materials** due to variability of loading was realized in 19th century. Most of the fractures of engineering structures are due to fatigue and the consequences are disastrous. How to envisage in what operational lifetime fatigue failure would occur, i.e. how to assess the fatigue life, has been one of the most important engineering problems for the last two centuries. *There was not any uniform and all-acknowledged solution under any loading including the general multiaxial case of non-proportional, non-cyclic and arbitrary wave-forms of the stress components.* According to the thesis, the lack of solution was since the fatigue life had not been searched by means of an integral of fatigue damage differentials. Such a possibility had not been revealed and exploited before but it can be practically implemented nowadays thanks to the computers.

The thesis opens a **new scientific research line** under the IDD abbreviation. The underlying statement is that only the universal mathematical way of the calculus from differentials to an integral can establish a uniform and all-acknowledged solution to the problem of fatigue life evaluation under any loading. *The new line proposed would re-direct a vast world-wide research experience, accumulated for nearly two centuries, into another course. The basic notion of that experience is loading cycle and therefore the hitherto existing approach is called Cycle Counting Approach (CCA).* In the thesis, another, new and radically different IDD approach is proposed: the basic, general notion is loading differential, while loading cycle remains as a particular notion, and the damage differentials per the separate loading differentials are integrated (summed). That such differentials are introduced for fatigue life assessment may have the same importance which the differentials introduced in the mathematics and exact sciences generally have: decisive.

The development of the IDD approach and creating unique IDD software has been done by the author only what has inevitably engaged a lot of time: about 30 years. Since everything proposed here is entirely original and without any existing analog, the colleagues in the world and in Bulgaria have taken an explainable position of waiting for results. Thus, the IDD work continued most of the time without any collaborators, nor any financial or other support. Nevertheless, the IDD approach has become well-known and discussed in the world.

What is said above, as well as the necessity of juxtaposition to the nearly two-century CCA experience, explain the inevitable fact of a comparatively large volume of the thesis: 353 pages (with expanded line spacing and with an IDD-software manual included). But the colleagues that will study it, as well as the members of the scientific jury, will quickly orientate themselves to the main points. To help them, this Summary, this Foreword and an extended peculiar Preview serve. Then, the Conclusion and the Contributions (presented separately) can be read. Afterwards, the details can be entered: the new notions, the mathematical instruments, the software created for practical application of author's IDD method, the verifications carried out and their results, and so on. The volume of these details has been compressed to an acceptable minimum. The IDD site cited above has been organized in a way as to also facilitate the study of the thesis. Besides, the site gives the IDD software as freeware. As well, the site gives the files involved in the sections of the thesis and in the verifications.

Eventually, this foreword hints that the thesis is expected to evoke great interest and opinions under considerable scientific responsibility on the part of the scientific jury members and the other colleagues. To all of them the author renders homage and his expectation of a just evaluation.

CONTENTS

(In the parentheses at the right margin, the page numbers of the thesis are put.
Some of the contents items of the thesis are not included in this summary.)

SYMBOLS	(11)
PREVIEW	8 (17)
A treatise on the differentials and integrals	8 (17)
Retrospection of IDD	8 (20)
To the attention of researchers studying the method proposed	(30)
CHAPTER 1. REVIEW ON EXISTING METHODS WITH CONCOMITANT ANALYSIS AND CONCLUSIONS IN THE VEIN OF IDD.	
GOAL AND TASKS OF THIS THESIS	14 (31)
1.1. Introductory notions, terms and symbols. Kinds of loading	14 (31)
1.2. Fatigue life under cyclic uniaxial or multiaxial proportional loading.	
<i>S-N</i> line	18 (40)
1.2.1. General notes	18 (40)
1.2.2. Influence of a static (mean) stress	20 (45)
1.2.3. About the <i>S-N</i> line under multiaxial state of stress	21 (48)
1.2.4. Composing an <i>S-N</i> line	22 (52)
1.3. Fatigue life under non-cyclic uniaxial or multiaxial proportional loading	22 (54)
1.3.1. Cycle counting (schematization). Miner rule.	
Amplitude spectrum	22 (54)
1.3.2. Cycle counting methods. Rain-flow method	23 (56)
1.3.3. Is it possible not to divide the loading into cycles and count them?	24 (60)
1.3.4. "History", "future", continuity, static level	25 (61)
1.3.5. Linear and non-linear damage accumulation	26 (63)
1.4. Fatigue life under multiaxial non-proportional loading	26 (66)
1.4.1. Reduction (decomposition) of the loading	26 (66)
1.4.2. Reduction to an equivalent stress	27 (68)
1.4.3. The concept of a critical plane and corresponding methods	27 (69)
1.4.4. Integral methods over the planes	28 (74)
1.4.5. Non-proportional $\sigma(t) \equiv \sigma_x(t)$ and $\tau(t) \equiv \tau_{xy}(t)$	29 (76)
1.4.6. Rotating bending with steady torsion	29 (78)
1.4.7. Non-proportional $\sigma_x(t)$ and $\sigma_y(t)$	30 (81)
1.5. Conclusions from the Review	30 (82)
1.6. The goal and tasks of this thesis	31 (84)
CHAPTER 2. IDD THEORY	32 (87)
2.1. Loading (stressing) differential	32 (87)
2.1.1. Stress differentials $d\sigma_x$, $d\sigma_y$ and $d\tau_{xy}$	32 (87)
2.1.2. Invariant loading differential ($d\sigma'$, $d\sigma''$, $d\tau$)	33 (88)
2.1.3. Geometrical form of the invariant loading differential ds	35 (92)
2.1.4. Components of the loading differential. Basic IDD types of loading. Resolution of the loading differential	37 (97)
2.2. Basic damage differentials and basic damage intensities	38 (101)
2.3. Determination of $R \equiv R_r$ and application of IDD to one value of k	39 (105)
2.3.1. Determination of $R(s)$ based on the Newton-Leibniz formula	39 (105)
2.3.2. The function $R(s)$ and $D(s)$ containing i^* divisor	40 (108)
2.3.3. <i>S-N</i> , <i>S-R</i> and <i>S-D</i> lines. 'Breaking' (impulse) mode and 'smooth' mode	41 (110)

2.3.4.	Numerical examples. Equation of 'bending' (smooth) $S-N$ line ...	43	(112)
2.3.5.	The opportunity for fatigue life computation without cycle counting, in impulse 'peak' and 'range' mode, and in smooth mode	43	(114)
2.3.6.	An example for the values of the i^* divisor with $s_m \neq 0$ and for the possibility to directly set $i^* = 2$	44	(118)
2.3.7.	Approximation of true damage intensity by the introduced symmetrical averaging intensity	44	(120)
2.4.	Determination of R_r in the whole $\sigma'-\sigma'$ plane. Concomitant issues	45	(122)
2.4.1.	Introducing lines of equal lives	45	(122)
2.4.2.	Taking static stresses (or R stress ratios) into consideration	46	(123)
2.4.3.	Symmetry towards the η axis	47	(127)
2.4.4.	Exchanging the values of the principal stresses (switching over the signs \pm). First, second and third condition	47	(129)
2.4.5.	Central symmetry and symmetry towards the ξ axis	49	(133)
2.4.6.	Composition of the lines of equal lives	49	(134)
2.4.7.	Determination of $N(s \equiv s_{\max})$ and $R_r(s)$ at any k	50	(136)
2.4.8.	Initial description of procedures in the <i>Ellipse</i> algorithm	51	(138)
2.5.	The R_c damage intensity in the whole $\sigma'-\sigma'$ plane. Concomitant issues	52	(139)
2.5.1.	Possible pure c -loading and determination of the basic R_c	52	(139)
2.5.2.	Introducing R_c -prototypes compared to R_r -prototypes	52	(144)
2.6.	The R_τ damage intensity in the whole $\sigma'-\sigma'$ plane. Concomitant issues	53	(147)
2.6.1.	The pure $d\tau$ -loading. The rotating disk of Findley et al.	53	(147)
2.6.2.	Stress analysis and another proposal for technical Implementation of the pure $d\tau$ -loading	54	(151)
2.6.3.	Possibility of determination of the basic R_τ	54	(155)
2.6.4.	Introducing R_τ -prototypes compared to the R_r -prototypes	55	(158)
2.6.5.	The pure $d\tau$ -loading as maximized case and other cases of 'weaker' loading	55	(159)
2.6.6.	Comparative fatigue life assessments in the cases considered (not in favor of the critical plane concept)	55	(163)
2.7.	The damage differential dD in the general case of combined loading. Versions of the IDD method	55	(164)
2.7.1.	Searching for an empirical formula for dD	55	(164)
2.7.2.	The first version (using \mathcal{E} , \mathcal{E}' and a single R -intensity)	56	(165)
2.7.3.	The generalization using three damage intensities	56	(166)
2.7.4.	Other more (simplified) versions. Additional notes	56	(168)
2.7.5.	IDD equation with the R_r -intensity and the factors f_c and f_τ . First and second practical category of non-proportional loadings	56	(171)
2.7.6.	Approximating the true damage intensity	57	(174)
2.7.7.	No-damage areas and lines that surround them	58	(174)
2.8.	IDD in statistical (probabilistic) interpretation under random loading	59	(178)
2.8.1.	Two-dimensional density of instantaneous values and computing the fatigue life based on it	59	(178)
2.8.2.	One-dimensional interpretation under random r -loading	59	(184)
2.9.	Interpolation for IDD	59	(185)
2.9.1.	The necessity of interpolation. A number for interpolation	59	(185)
2.9.2.	Trigonometric interpolation	59	(187)
2.9.3.	Cubic-spline interpolation	60	(189)
2.10.	Conclusions	60	(191)

CHAPTER 3. SOFTWARE AND VERIFICATIONS OF IDD UNDER A SINGLE OSCILLOGRAM	61	(193)
3.1. The <i>Integral</i> algorithm	61	(193)
3.1.1. An oscillogram as an example	61	(193)
3.1.2. Algorithmic IDD equation of the fatigue life	62	(194)
3.1.3. The <i>Integral</i> computer program. Demos	62	(195)
3.2. Comparison with CCA (Cycle Counting Approach)	63	(198)
3.2.1. Preliminary analysis	63	(198)
3.2.2. Comparison to the rain-flow method under the same oscillogram	63	(200)
3.2.3. The smooth $S-N$ line and its R prototype	63	(205)
3.2.4. Examples under a non-zero static level and comparison	64	(207)
3.3. Real tests and comparison (under zero static level)	64	(212)
3.3.1. Experimental oscillograms and $S-N$ line	64	(212)
3.3.2. Experimental and computed lives	64	(215)
3.4. Conclusions	66	(220)
CHAPTER 4. THE <i>ELLIPSE</i> SOFTWARE	66	(221)
4.1. The more important mathematical and algorithmic details	66	(221)
4.1.1. Intensities and input oscillogram involved, and a 'Code'	66	(221)
4.1.2. The transformation of the variant elements into invariant ones ..	66	(223)
4.1.3. The angular third switchover condition and determination of the $\alpha'(t)$ function	67	(227)
4.1.4. Radial third switchover condition and concomitant dividing variant elements into two sub-elements. Continuity or discontinuity over the η axis	67	(236)
4.1.5. Solution of the problem of dividing into two sub-elements	68	(241)
4.1.6. Elements Δs_r close to O . The current point falling into the L area. About the impulse mode		(244)
4.1.7. Details about the current elliptic equation	68	(246)
4.1.8. Analysis on the possibility to obtain and solve the current elliptic equation	68	(248)
4.1.9. Solving the current elliptic equation and computing the damage intensity	68	(252)
4.1.10. The graph mode	68	(254)
4.2. The <i>EllipseT</i> program	70	(258)
4.2.1. Demo of entering the leading data		(259)
4.2.2. Demo of entering the current data and obtaining the life		(262)
4.2.3. Demo of the graph mode, etc.		(266)
4.2.4. The trigonometric polynomial. Demos regarding the influence of n_i		(268)
4.2.5. Demos of disconnected and connected Δs_{xy} elements, $\Delta \tau$ -loading, etc.		(270)
4.3. The cubic-spline interpolation. The program <i>EllipseS</i> (and <i>EllipseC</i>) ...	70	(273)
4.3.1. Some mathematical and algorithmic details		(273)
4.3.2. Demos with <i>EllipseS</i> (similar to sections 4.2.1 – 4.2.3)	70	(275)
4.3.3. Demos with <i>EllipseS</i> (similar to Section 4.2.5)		(276)
4.3.4. The <i>EllipseC</i> program	71	(277)
4.4. Conclusions	71	(278)
CHAPTER 5. IDD VERIFICATIONS UNDER NON-PROPORTIONAL LOADINGS OF THE FIRST PRACTICAL CATEGORY	71	(279)
5.1. Strategy of the verifications	71	(279)
5.2. Initial Adaptation (0) for determination of the IDD parameters		

(using data of Timshin and Hazanov)	72	(280)
5.2.1. Experimental data	72	(280)
5.2.2. Adaptation according to the simplified IDD version with a single R -intensity	73	(283)
5.2.3. Adaptation according to the main IDD version with the three intensities	74	(285)
5.3. Verification (1) (using data of Neugebauer)	75	(289)
5.3.1. Experimental data	75	(289)
5.3.2. Computing the IDD lives under the non-proportional loadings	76	(291)
5.3.3. Additional and conclusive notes	76	(292)
5.4. Verification (2) (using data of Simbürger)	76	(294)
5.4.1. Experimental data	76	(294)
5.4.2. Composing the input files	77	(296)
5.4.3. Verification results and conclusions	78	(297)
5.4.4. A check of influence of systematic error	78	(298)
5.5. Verification (3) (using data of Atzori et al.)	78	(300)
5.5.1. Experimental data and input prototypes	78	(300)
5.5.2. The data under combined 90^0 -out-of-phase loading	78	(301)
5.5.3. Computation of the lives and conclusions	78	(302)
5.6. Verification (4) (using data of Störzel et al.)	79	(303)
5.6.1. Laserbeam welded tube-tube specimens and local stresses in them	79	(303)
5.6.2. Experimental data for input prototypes	80	(305)
5.6.3. 90^0 -out-of-phase experimental and computed lives.....	81	(307)
5.6.4. 45^0 -out-of-phase experimental and computed lives.....	81	(310)
5.6.5. Conclusions after Verification (4)	81	(311)
5.7. Verification (5) (using data of Sonsino)	82	(312)
5.7.1. Experimental data and composing the R_r -prototypes	82	(312)
5.7.2. Computation of 'in-phase' lives	82	(314)
5.7.3. 90^0 -out-of-phase experimental and computed lives, and conclusions	83	(315)
5.8. Verification (6) (using data of Stoychev)	83	(317)
5.8.1. Experimental data under rotating bending with constant torsion	83	(317)
5.8.2. Forming the L-files and C-files	84	(318)
5.8.3. Graph mode illustration and additional considerations	84	(321)
5.8.4. Computed lives. Conclusions	84	(322)
5.9. Conclusions from Chapter 5	84	(323)
5.9.1. IDD N_{cmp} - N_{exp} diagram	84	(323)
5.9.2. Empirical data bank of the IDD parameters and conclusions	85	(324)
CHAPTER 6. NECESSITY AND POSSIBILITY FOR APPLICATION OF IDD TO MACHINES AND TECHNICAL EQUIPMENT IN THE FOREST INDUSTRY	87	(327)
6.1. Registration of this thesis to only one accredited scientific speciality	87	(327)
6.2. First example [33]	87	(328)
6.2.1. Circular shaft. Kinematics of cutting	87	(328)
6.2.2. Cutting forces on the teeth	88	(331)
6.2.3. Approximate expectations of the normal and shear stress oscillograms	88	(332)
6.2.4. Conclusion	91	(336)
6.3. Second example	91	(336)
6.3.1. Band-saw blade	91	(336)

6.3.2. Calculation scheme	91	(338)
6.3.3. Expectations of the tensile stress oscillogram. Conclusion	92	(339)
CONCLUSION	93	(341)
REFERENCES		(343)

5.9. Conclusions from Chapter 5

5.9.1. IDD $N_{\text{cmp}}\text{-}N_{\text{exp}}$ diagram

In Fig. 5.9.1-1, there are placed in all 49 values of N_{exp} and corresponding 49 values of N_{cmp} from the adaptation (0) and the verifications (1) – (6). Thus, Fig. 5.9.1-1 presents the IDD $N_{\text{cmp}}\text{-}N_{\text{exp}}$ diagram as a conclusion from Chapter 5. Three of the N_{exp} values are considered as ‘greater than’ for the availability of run-outs (survived specimens). For the rest

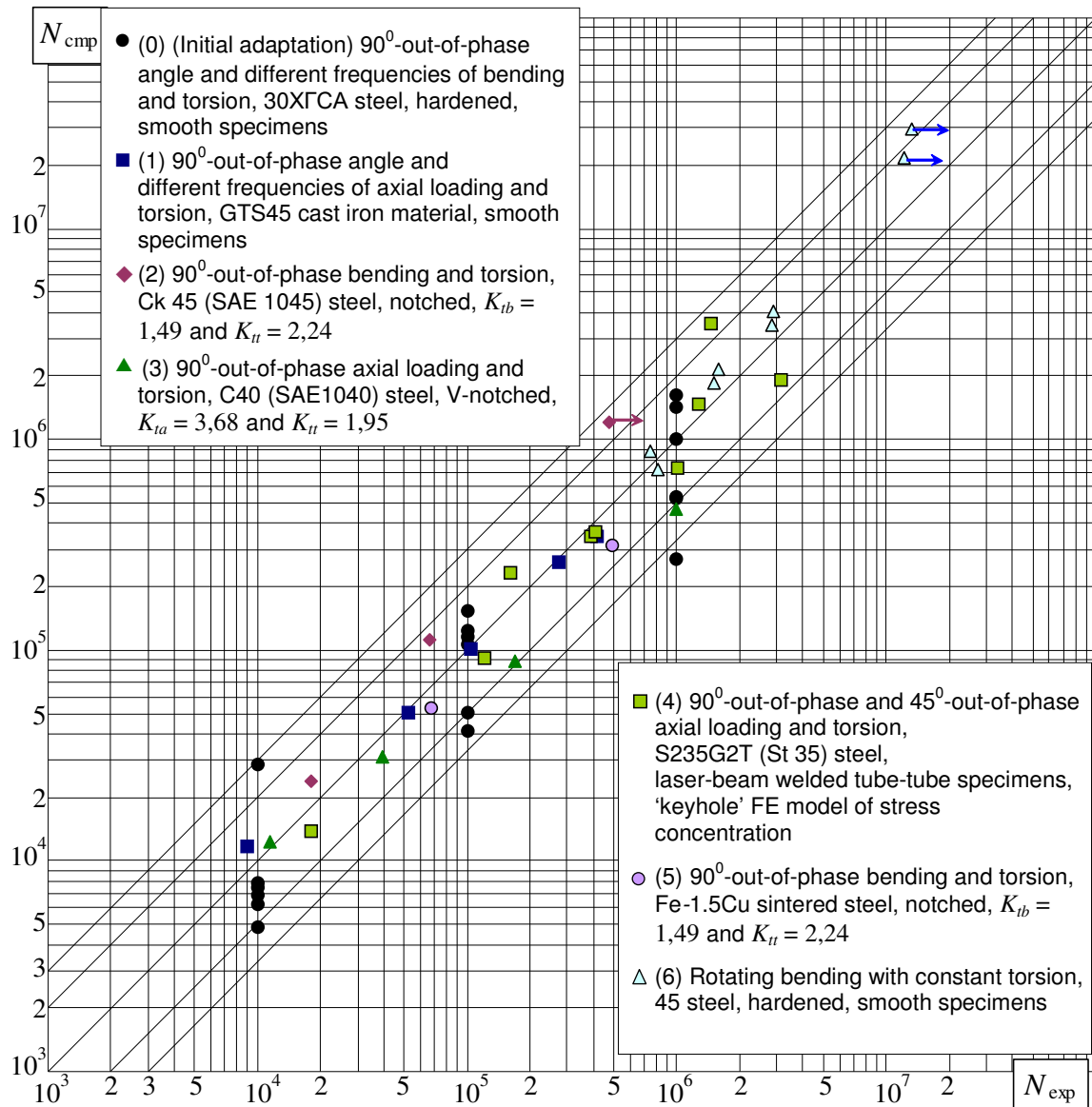


Fig. 5.9.1-1. IDD N_{cmp} - N_{exp} diagram from the (0) initial adaptation and the (1) – (6) verifications (the N_{cmp}/N_{exp} ratios have 1,02 average and 0,52 standard deviation from the average)

46 N_{cmp}/N_{exp} ratios the statistical data (Ch5.xls file) are valid, as follow.

The average is 1,02: an extremely good result. The standard deviation (from the 1,02 average) is 0,52: a very successful result. It means that in future similar IDD applications N_{cmp} will be expected to appear most probably between $1,54^{-1}N_{exp} = 0,65N_{exp}$ and $1,54N_{exp}$. The greatest deviations of N_{cmp}/N_{exp} are Min $0,27 = 3,68^{-1}$ and Max 2,86. They have both appeared in the initial adaptation whereas in the verifications (1) – (6) Min is $0,47 = 2,14^{-1}$ and Max is 2,45. By the way, the average of N_{cmp}/N_{exp} from the verifications (1) – (6) is 1,05 and the standard deviation is 0,41.

5.9.2. Empirical data bank of the IDD parameters and conclusions

So far, based on the thesis (and the latest studies being placed in Volume II), the following empirical data bank (Table 5.9.2-1) has been formed for the first practical category of non-proportional loadings. The bank is, of course, initial and uncompleted but is a good basis for next IDD fatigue life evaluations.

Dr. Papuga has also applied other methods but so far their computed lives results significantly yield to the IDD results.

The numbers N_c and N_τ are left with some indefiniteness in Table 5.9.2-1. In fact, the areas L_c and L_τ are not strictly given. This corresponds to the opinion, already mentioned, that the notion of fatigue limit contains some uncertainty and fictitiousness especially under arbitrary non-proportional loadings. What only looks for sure is that the L_c and L_τ lines should be more inward to the coordinate origin than the L_r (or L_l) line. This can be ensured even by the equalities $N_c = N_\tau = N_r$ but only if $f_c > 1$ and $f_\tau > 1$. Otherwise, in case f_c and f_τ near 1, then $N_c > N_r$ and $N_\tau > N_r$ should be set.

As to trajectories which go farther out of the L_r area, setting N_c and N_τ to be or not to be greater than N_r does not seriously influence N_{cmp} . However, if the trajectory does not go out of the L_r area, then the selection of N_c and N_τ to be greater than N_r becomes important. As already discussed, if a trial increase of N_c and N_τ sharply changes N_{cmp} , then stronger inequalities $N_c > N_r$ and $N_\tau > N_r$ should be preferred. Generally speaking, as a tentative recommendation, N_c and N_τ should be selected in the order of $10N_r$ with possible error in favour of safety.

It is to additionally note that fatigue life assessment methods are also proposed by a part of the authors of the experimental data used in the IDD verifications done. None of those methods has been simultaneously applied to all the experimental data of the six IDD verifications. If such a trial is done, it will be seen that the conclusions from the review of the existing methods (Section 1.5) are confirmed: too various methods, proved only in specific loading cases, incompatible (or inapplicable) and conflicting to each other in all the cases.

CONCLUSION

(Concluding notes)

The mechanism of forming the fatigue life is too complicated and inscrutable. This provides a possibility to every researcher to propose his own CCA method (model) for fatigue life assessment which contains some partial truth and is experimentally confirmed in some limited scope. Another researcher, under different experimental data and different loadings, does not find confirmation of the previous method and, in his turn, also proposes his own model. After all, there is not any uniform, all-acknowledged and universal CCA method but there are many disputable methods comparing to each other, each one with its partial truth and that is why they tolerate each other.

Now, something radically different is proposed: not to have loading cycle as the basic notion but to have it as a particular notion, and, instead of searching for disputable cycles, to follow the indisputable differentials (Fig. 1.1-3b) of any loading and directly compute the damage differentials per the loading differentials.

Thus, it is not simply about a serial new method in expectation of a tolerant attitude. Now, a united skeptical or negative reaction is possible: considering that many thousands of fatigue life researchers in the world had searched for cycles in every loading, the IDD concept and the author's IDD method could make a lot of opponents. They could even state that IDD rejects all the accumulated fatigue life knowledge built on the basis of the notion of loading cycle. On this occasion it is to pay attention again to the contributions 2.3 and 13.4: IDD rejects nothing of the existing knowledge. The thesis has entirely been built on the basis of the existing knowledge and the main idea is to use it in another way.

It is understandable that the colleagues would express skepticism and jealousy after they have devoted their investigations and careers to CCA and received acknowledgements and degrees for that. It is understandable that they would look for weak points of the IDD

concept and of the author's IDD method, and would raise controversial questions since the complicated and inscrutable mechanism of forming the fatigue life leaves a large place for a lot of disputation.

However, there is a possibility which leaves no place for disputation on whether IDD should be acknowledged or not, as follows.

Let any IDD opponent verify any other fatigue life evaluation method by using the same experimental data files used also by the author in the IDD verifications done in Chapter 5. Let the verification in Chapter 3 be also added. Let the verification or verifications continuing in Volume II be added, as well. All the experimental data are not of the IDD author but of other authors and therefore a partial selection of one's own experimental data is excluded. And if the IDD opponent proves that the other method is always applicable in all the mentioned cases and categorically excels the IDD method, then the disputation ends: the IDD method should withdraw. Moreover, the other method will prove to be that missed one which can claim for general validity now. But if the other method yields, the disputation ends again: the IDD method should be given the right of way. Moreover, resetting and canalizing the world investigations to IDD should be recommended.

If the other method and the IDD method turn out to be approximately tantamount, again the IDD method should be given the right of way to continue comparing and proving itself in next and next verification (Volume II). With that, the oscillograms should purposefully be diversified as much as possible: to be of both the first and second practical category of non-proportional loadings, and of mixed loading with various trajectory ratios t_r , t_c and t_{τ} , and of various trajectory's forms, and of both cyclic and non-cyclic loadings, and of both deterministic and random loadings, and of pure r -loadings, pure c -loadings, pure $d\tau$ -loadings, and so on.

Hereby the author closes and lets the colleagues and the honorable scientific jury members take the scientific responsibility for the evaluation of the thesis.